Homework #8

Introduction to physical oceanography

*Numerics:* You saw in class how to solve the 1d advection diffusion equation numerically:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = \kappa\frac{\partial^2 C}{\partial x^2}.$$

Each of the following questions starts from the program for solving this equation as posted on the course home page.

1. (a) plot the solution as function of time in the middle of the domain for a long enough time such that the Gaussian peak re-enters the domain 3-4 times. Plot the solution also as a contour plot which is a function of time and space. look at the animation the program provides to get a feeling for what happens, and describe the characteristics of the solution.

   (b) set the Robert filter coefficient to zero and plot the solution in the middle of the domain as function of time. explain how the results depend on the value of the Robert filter coefficient. do the same when the advecting velocity is 5, 10, and 20.

   (c) reduce the diffusion coefficient as much as possible without causing the program to crash (i.e., give unreasonable results). what is the effect on the solution?

   (d) increase the time step until the program crashes. then half the time step, and plot the solution in the middle of the domain as function of time only. half again and plot again. are the solutions different for different time steps? are they getting similar after a few halvings of the time step? why?

2. add to the equation a radioactive decay term, such that the equation is now

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = \kappa\frac{\partial^2 C}{\partial x^2} - \lambda C.$$

   set $\lambda$ to $10^{-8}$, make the appropriate change to the program and explain what you have done to implement this change to the finite difference formulation. Reduce the diffusion coefficient $\kappa$ as much as possible, look at the animation and plot the solution in the middle of the domain. how is the behavior different from the case where there was no decay and there was larger diffusion?

3. Increase the number of spacial points to nx=200. Change the initial conditions to a sine wave plus a constant, as function of space (making sure the initial concentration is never negative). try two cases: one with 10 wave lengths in the domain, and one with only 3. For each case, run the program with both $\lambda = 10^{-8}$ and very small diffusion $\kappa$, as well as with $\lambda = 0$ and standard diffusion. plot the solution as function of space after a sufficiently long time that one can see a reduction ot the wave amplitude by a factor of 3 in the standard run. How is the behavior different in these four cases that you have looked at in this question? Explain the results in terms of scale-selective diffusion vs non scale-selective diffusion (see similar discussion in class on scale-selective and non scale-selective dissipation).