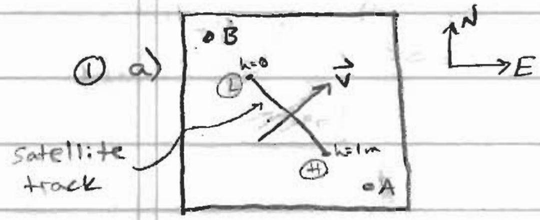
EPS 131 Problem set #3 SOLUTIONS

Ian Eisenman

1 a)



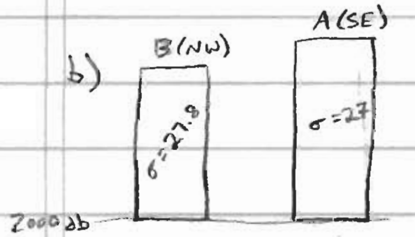
$-fv = -\frac{1}{\rho_0}\frac{\partial p}{\partial x}$ and $fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y}$, but the pressure gradient is not purely meridional (N-S) or zonal (E-W). As vectors,

$$f\,\hat{k} \times \vec{u} = -\frac{1}{\rho_0}\nabla p \Rightarrow f|\vec{u}| = \frac{1}{\rho_0}|\nabla p|$$

with $\frac{\partial p}{\partial z} = -\rho_0 g \Rightarrow p(x,y,z) = \rho_0 g\,(h(x,y) - z)$ (hydrostatic approx),

$$|\vec{u}| = \frac{g}{f}|\nabla h| \simeq \frac{g}{f}\frac{\Delta h}{\Delta \ell} = \frac{g}{2\Omega \sin\theta}\,\frac{1\,\text{m}}{200\,\text{km}} = \boxed{0.59\ \text{m/s NE}}$$ (direction from picture)

b)



Column B is shorter and denser, but both columns above 2000 db have same weight (because 2000 db is level surface).

Let $h=0$ at column B. Then the depth is

$$p = 2000\,\text{db} = \rho_0 g\,(h - z) = -\rho_B g z \Rightarrow z = -\frac{2000\,\text{db}}{\rho_0 g}$$

To find h at column A, use

$$p = 2000\,\text{db} = \rho_A g\,(h - z) \Rightarrow h_A = z + \frac{2000\,\text{db}}{\rho_A g} = \frac{2000\,\text{db}}{g}\left(\frac{1}{\rho_A} - \frac{1}{\rho_B}\right) = 1.55\,\text{m}$$

$$\text{slope} = \frac{\Delta h}{\Delta \ell} = \frac{1.55\,\text{m}}{250\,\text{km}} = \boxed{6.2 \cdot 10^{-6}}$$

$$|\vec{u}| = \frac{g}{f}|\nabla h| \simeq \frac{g}{f}\frac{\Delta h}{\Delta \ell} = \boxed{0.70\ \text{m/s NE}}$$ (again, direction from picture)

c) The satellite data gives us the actual sea surface height from which we can calculate geostrophic surface currents. The density measurements allow us to find how the geostrophic current varies with depth (thermal wind equation), but we need to assume a level of no motion to integrate from for surface currents. In this problem, the actual level of no motion is at a slightly different height than 2000 db. We can find the velocity at 2000 db from the pressure gradient.

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① c) cont'd

Let the height above column B be $h=0$. Above column A, then, $h = \frac{1\,m}{200km} \cdot 250km = 1.25\,m$

$p_B - p_A = g(\rho_B(-z) - \rho_A(h-z)) = g((\rho_B - \rho_A)(-z) - \rho_A h) = g((0.8\frac{kg}{m^3})(2000m) - \rho_A(1.25m))$

$= 3.1 \cdot 10^3 \frac{N}{m^2}$

$|\vec{u}| = \frac{1}{f\rho_0}|\nabla p| \simeq \frac{1}{f\rho_A} \frac{(p_B - p_A)}{250\,km} = \boxed{0.14\,m/s}$

Since $p_B > p_A$, velocity is $\boxed{SW}$.

d) Since $\rho_B > \rho_A$, $\boxed{\text{station A}}$ has warmer (less dense) water, which is to the $\boxed{\text{left}}$ of the current at 2000 db.

② $E = mc\Delta T$

$\Delta T = \frac{E}{mc} = \frac{P\Delta t}{V\rho c} = \frac{P}{A}\frac{\Delta t}{\Delta z \rho c}$

V is volume $= \Delta z \cdot A$, $\Delta z = 50m$; $\frac{P}{A}$ = power/area $= 35\frac{W}{m^2} = 35\frac{J}{s\,m^2}$

$\Delta t = 1$ year; $\rho = 1027\frac{kg}{m^3}$; c = heat capacity of water $= 4.184\frac{J}{g\,°C}$

↳ $\boxed{\Delta T = 5°C}$

(3) - station is in south tropical Atlantic, off the coast of Angola (Africa west coast), east of mid Atlantic ridge.

- see figure below.
- intersections (water types):

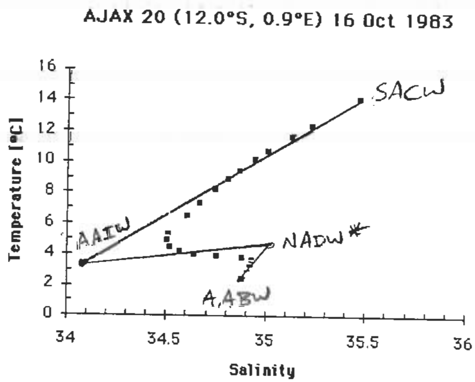
  Antarctic Bottom Water (AABW): 1°C, σ = 34.8

  North Atlantic Deep Water (NADW): 4°C, σ = 35

  Antarctic Intermediate Water (AAIW): 3.2°C, σ = 34.5

  South Atlantic Central Water (SACW): 14°C, σ = 35.5

- These observations fit excellently with the idea of water mass mixing. Note that the data points cut corners in the figure below because more than 2 water types are mixing (cf., Pickard and Emery, p. 145).



* or perhaps circumpolar water.