Homework #3
Nonlinear dynamics and chaos

(a) Linearized 2d systems:

Classify the stability of the fixed points of the following systems by solving for their eigenvalues/ vectors and plotting the vector field in the phase plane. If the eigenvectors are real, plot them in phase space. Can use the quiver function of Matlab for the plot.

$$\dot{x} = x - y; \qquad \dot{y} = x + y \tag{1}$$
$$\dot{x} = 5x + 2y; \qquad \dot{y} = -17x - 5y \tag{2}$$
$$\dot{x} = 5x + 10y; \qquad \dot{y} = -x - y \tag{3}$$

(b) Nonlinear 2d systems:

Find the fixed points, classify them, sketch neighboring trajectories, and try to fill in the rest of the phase space portrait:

$$\dot{x} = x - y; \qquad \dot{y} = x^2 - 4 \tag{4}$$
$$\dot{x} = \sin y; \qquad \dot{y} = \cos x \tag{5}$$
$$\dot{x} = xy - 1; \qquad \dot{y} = x - y^3 \tag{6}$$
$$\dot{x} = xy; \qquad \dot{y} = x^2 - y \qquad \text{beware, linearization fails here. why?} \tag{7}$$

(c) Phase locking:

Consider the circle map:

$$\theta_{n+1} = \theta_n + \Omega - \frac{K}{2\pi}\sin(2\pi\theta_n) \qquad \textit{mod } 1 \tag{8}$$

a Matlab program for the circle map is on the course home page.

1. Find a period 2 solution ($p/q = 1/2$) for $K > 1/2$, $\Omega \neq 1/2$.
   (a) Show that the solution does not depend on the initial conditions θ_0 by iterating the map from 2 different initial conditions, converging to the same period-2 solution.
   (b) Find another ($p/q = 1/2$) solution for different K, Ω, with again $K > 1/2$, $\Omega \neq 1/2$, show that it is different from the previous one although it has the same period

2. Find a $p/q = 0/1$ solution for $\Omega \neq 0$, $K > 1/2$. Describe the behavior of this solution as function of n. Do the same for a $p/q = 1/1$ solution for $\Omega \neq 1$, $K > 1/2$.

3. find a solution $p/q = 3/4$ for $K > 1/2$.

4. try $K > 1$. What happens?